**Essay**

**Leonardo da Vinci and the search for order in neuroscience**

**Gustavo Deco**1,2,3,4, **Martin Kemp**5, and **Morten L. Kringelbach**6,7,8

Finding order in disorder is a hallmark of science and art. In the time of Leonardo da Vinci, the schism between science and art had yet to arise. In fact, Leonardo freely used scientific methods for his art and vice versa; for example, when he used his observations of turbulent, whirling water to guide his artistic imagination. Half a millennium later, a cornerstone of modern biology is the continuing search for order in dynamic processes. In neuroscience, the search has focussed on understanding complex spacetime brain dynamics. Recently, turbulence has been shown to be a guiding principle underlying the necessary information processing, supporting Leonardo’s search for order in disorder. Here, we argue that Leonardo’s seminal insights have ongoing relevance for modern neuroscience.

Let no-one who is not a mathematician read my principles
Leonardo da Vinci1

Leonardo da Vinci died over 500 years ago. Why does he feature in an essay on neuroscience? He is not featuring here because he was a ‘man ahead of his time’, as the cliché goes. Nor is he serving to provide a cultural gloss for modern scientific discoveries. And establishing his direct influence on the course of science is not our primary aim. Rather, we are looking at the deeper cognitive level of ‘structural intuitions’ in which instinctive sensing of underlying patterns provide common starting points for certain kinds of art and science2. This concept has recently been extended from morphology to dynamics and the discerning of order in chaotic and non-chaotic systems3,4. As it happens, theories of self-organised criticality can be applied to the physical processes in varied media that artists knowingly instigate to create unforecastable order (for an example, see Kemp5).

Dynamic order is apparent not only in those things that are being observed in external turbulence, but also in the thought processes themselves in which Leonardo’s non-linear and entwined modes of thinking and representation assumed what we can describe as a turbulent mode of operation. This mode is one of the general characteristics of creative thought. What Leonardo does is to use his powers of visualisation and graphic skills to express external and internal turbulence at supreme levels that still speak to modern science.

Leonardo da Vinci was constantly looking for novel ways to understand how the complexity of Nature arose from a set of pre-Newtonian laws of dynamics (for his most developed discussions of water, see the Codex Leicester9). Representing complexity of motion was extremely difficult, not least when he tried to draw the incessant movements of intersecting bodies of water. Given that it was impossible 500 years ago to freeze this movement in time, it required a great visualizer to develop a deep understanding of the underlying ordered mechanisms that create the seeming disorder. The quotation at the head of this essay comes from a page on which he was visualizing turbulent blood flow in heart valves. He threw mathematics at the complexity of fluids in motion in a pioneering way.

Leonardo was fully aware of how water in a river can flow in an ordered layered fashion in what is now called laminar flow. He also studied how a sudden obstacle like a branch in a river can create turbulent flow, resulting in different types of vortices. Laminar flow is beautiful but boring and not particularly useful, while turbulent flow is seemingly chaotic but deeply useful. For example, when cooking, laminar flow does not allow for proper mixing of the ingredients, but when you introduce stirring and thus turbulence, much more efficient mixing occurs.

Merging art and science into a seamless whole, Leonardo was able to discern and represent underlying orders in seemingly disordered natural phenomena. His extended analyses of the behaviour of water combine mathematical theories of motion, as understood at the time, with acute observation. His quest to embrace complexity is reflected in how he used vernacular Italian to describe his observations. At one point he listed 68 terms that might describe the varieties of fluid motion and its many effects (for images and transcriptions, see https://www.leonardodigitale.com, Paris, Institut de France, MS I 72r-71r). The best collective word for all the motions was ‘turbolenza’ from the Latin word for ‘crowdiness’ or disturbed, in this way capturing the disorder of a turbulent regime no less than the agitation of fluids.

From the Codex Leicester, owned by Bill Gates, we know that Leonardo used scientific ‘laboratory experiments’ to tease out the ordering principles at work9. He described how to capture the formation, interaction and destruction of eddies of whirling water and bubbles of air by constructing a tank with sides made of glass. He proposed to infuse the water with seeds of panic grass (Panicum spp.) to track the eddies in complex interaction (Figure 1). This ingenious set-up also allowed him to study the effects of wind on the surface of water and laminar flow in deeper layers. For a modern treatment of his setup and the underlying physics, see Monaghan and Kajtar9.

Based on these and other observations he wrote on the sheet illustrated in Figure 2:

Observe the motion of the surface of the water, which is similar to that of hair, which has two motions, of which one is caused by the weight of the water the other by the course of the curls. In the same way, the water has curling vortices, one part of which is due to the principal course the other to the incident and reflected motion10.

Leonardo envisaged that the motion of a body or fluid should complete its due course according to the impetus impressed in it, whether its course is straight or deflected:

 Universally, everything desires to maintain its natural state. So moving water strives to sustain the power of the cause of its motion, and, if it finds opposition to its path, it completes the span of the course it has commenced by a circular and
Leonardo’s engagement with turbulence went beyond observable physical phenomena. It extended to his processes of thought. Indeed, the motion of water can be said to serve as ‘the mirror of his mind’ (see chapter 2 of Laurenza and Kemp). Turbulence permeated the restless motions of his imagination as it cascaded from one topic to another. This mental turbulence is vividly apparent when we look at how Leonardo invented compositions. Figure 3 demonstrates how he developed an entirely novel ‘brainstorming’ manner of drawing in which intertwined forms emerge and vanish like vortices in fluids. The illustrated sheet of experimental drawings for the Madonna, Child, St. Anne, St. John and a Lamb includes a water wheel! (London, British Museum, 1875-6-12-17, see Kemp and Barone). His creative processes are what we would now call non-linear: imagination (fantasia) allowed him to assemble ‘monsters’ from the component parts of different animals. He cultivated inventive powers that worked in a fluid and turbulent manner — more openly than anyone had previously done. The motions of water justly serve as a simile for the motions of his thought.

Leonardo’s vision of turbulence crossed every scale in nature, ranging from the massive storms in his drawings of catastrophic deluges to the tiny motions of eddies and bubbles in lesser bodies of water. He noted that:

Eddies with large revolutions are uncommon in the flow of rivers, and the small eddies are almost numberless; and large things are rotated only by large eddies and not by small ones, and small things are turned by both small eddies and large (Paris, Institut de France MS F 3r). Leonardo’s characterisation of eddies at varied scales remarkably predates the seminal observations in verse by the English polymath Lewis Fry Richardson (1881–1953), pioneer of the mathematical weather forecasting, who described the important turbulent energy cascade principle. As shown by Leonardo, there are differently sized vortices or eddies in a fluid, where each eddy corresponds to a rotational movement. The interactions between large and smaller eddies interchange energy, in the form of velocity or kinetic energy; this is called the energy cascade and transfers energy across scales, which roughly correspond to the size of different eddies.

This energy cascade was described in a humorous verse by Richardson: “Big whirls have little whirls / That feed on their velocity, / And little whirls have lesser whirls / And so on to viscosity ...”, a play on Siphonaptera, the taxonomic order of fleas, a brief poem by Augustus De Morgan, rewording Jonathan Swift: “Great fleas have little fleas upon their backs to bite ‘em; And little fleas have lesser fleas, and so ad infinitum”.

Figure 1. Leonardo da Vinci: experimental studies of turbulence in water. Leonardo da Vinci, Studies of Turbulence: rectangular obstacles in flowing water; and water pouring from an aperture into his experimental tank. Windsor Castle, Royal Library, 12660V. Image from Royal Collection Trust © Her Majesty Queen Elizabeth II 2021.
In fact, the scientific study of turbulence is one of the great triumphs (and problems) of modern physics. On his death bed, the German physicist Werner Heisenberg reportedly said, “When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first”. While famous for his work in quantum physics, Heisenberg finally discovered the fundamental statistical rules of turbulence in 1946, while interned in England after the war.

Unbeknownst to Heisenberg, however, the Russian mathematician Andrey Kolmogorov had already published this result in his ground-breaking phenomenological theory of turbulence\textsuperscript{15,16}. This highly influential theory demonstrates a fundamental power scaling law, revealing the key underlying mechanisms of fluid dynamics, namely the energy cascades that balance kinetics and viscous dissipation. This spatial power-scaling law is a hallmark of turbulence and provides a mathematical description of Richardson’s earlier concept of cascaded eddies\textsuperscript{17}. This correlates remarkably with Leonardo’s observation that the constriction of circumference towards the centre of the vortex is more rapid than the diminution of the water’s impetus, which is why the water revolves faster near the centre (Milan, Biblioteca Ambrosiana, Codice atlantico 813r\textsuperscript{18}). As so often with Leonardo, initial observation precedes mathematical theory. In turbulence, as we now know, this phenomenon comes about through the build-up of kinetic energy in the smaller eddies which translates into higher velocities.

Kolmogorov’s approach was very successful in overcoming the severe limitations of the then prevailing description of turbulence, which tried to describe the movement of each particle of the fluid mathematically. These fundamental movements are described by the Navier-Stokes equations, named after the French engineer Claude-Louis Navier and the British mathematician George Gabriel Stokes\textsuperscript{19,20}. The idea is to use these equations on the microscopic elements of the fluid to infer or construct the macroscopic laws governing the whole fluid. This constructivist approach of studying fluid dynamics at the microscopic level had comparably little success given the large computational power needed, which was not available in those days.

There are strong parallels to the way that the scientists have tried — and failed — to describe the macroscopic behaviour of the brain by modelling each microscopic neuron with the Hodgkin–Huxley equations\textsuperscript{21}. These were named after the Nobel prizewinning British physiologists Alan Hodgkin and Andrew Huxley, who described the activity of neurons by modelling the ionic mechanisms underlying the initiation and propagation of activity in the squid giant axon.

Again, however, there are computational problems with such a pure constructivist, bottom-up approach, which is inappropriate for explaining most complex phenomena, including fluid and whole-brain brain dynamics. Instead, the study of turbulence is better described by the statistical approach started by
Kolmogorov’s vital insight. On an abstract level, Kolmogorov’s approach is a way to discover order in disorder, which is, of course, exactly the same approach used by Leonardo over 500 years ago when he tried to characterise the ordered vortex configurations of crowded, disordered turbulent flows in fluids.

Similar to fluids, it is clear that brain activity should be described statistically directly at the macroscopic level. The idea that interrelated disciplines of turbulence, chaos theory and criticality can be relevant for describing chaotic patterns in biology comes from the seminal books of Arthur Winfree\textsuperscript{22}, Gyorgy Buzsaki\textsuperscript{23} and Per Bak\textsuperscript{24}. Their work shows the relevance of using these tools to describe the complexity of the biological patterns in nature. Take for example the measurements offered by magnetoencephalography used to measure brain activity in humans, particularly where it was used to search for consistent evidence for turbulent (chaotic) dynamics marked by intermittent turbulent eddies\textsuperscript{25}. Furthermore, at the local level, where local field potentials have been used to measure signals in the rat hippocampus, important research has been carried out showing traces of turbulence dynamics in this brain region\textsuperscript{26,27}. On a more general level, there has been speculation about how best to use the tools of turbulence to gain new insight into biological phenomena such as the heart\textsuperscript{28} and brain\textsuperscript{29,30}.

A turbulent system thus links Kolmogorov’s energy cascade with dissipation which can be applied directly to neuroscience, as suggested by Buzsaki\textsuperscript{23} who wrote “... perturbations of slow frequencies cause a cascade of energy dissipation at all frequency scales...”. More generally, Per Bak and colleagues\textsuperscript{31} wrote that “long-wavelength perturbations cause a cascade of energy dissipation on all length scales, which is the main characteristic of turbulence”. Initially, this research led to an infatuation with the phenomena of ‘self-organised criticality’ and ‘neural avalanches’. But turbulence remains a more fundamental description than these heuristics of activity distribution.

From the beginning neuroscience was partial to the metaphor of turbulence, but crucially without applying the powerful tools of Kolmogorov and Kuramoto’s turbulence theories to the data. Donald Hebb spoke of ‘cell assembly’ conceived as small recurrent neural networks mimicking a bucket of water, in this way linking back to Leonardo — and of course to Hebb’s postdoctoral adviser Karl Lashley who also proposed, in a beautiful paper\textsuperscript{32}, that concepts and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Leonardo da Vinci: turbulence as a tool for composition. (A) Leonardo da Vinci, Studies for the ‘Virgin, Child, St. Anne and St. John the Baptist’, with water wheels and a dam, London, British Museum. (B) Leonardo da Vinci, Sketch of a Dragon assembled from various animals. Windsor, Royal Library, 12360. Image from Royal Collection Trust © Her Majesty Queen Elizabeth II 2021.}
\end{figure}
mental processing work like water. This idea has seen a recent resurgence with novel incarnations of recurrent neural networks, like echo-state networks and liquid state machines.

Recently, we were able to use the tools developed in turbulence to describe how whole-brain activity exhibits turbulence. This is not surprising given that in order to survive in a complex world a brain has to mix a large amount of information across space and time. This is exactly what turbulence is good for. It also provides a general principle of order in the seemingly disordered and complex brain dynamics.

Once we had shown this ordering principle, we started to explore how brain activity can be characterised, not by microscopic equations but by coupled activity between brain areas. In fact, the simplest way to describe coupled activity is through oscillators. A very simple example of oscillator is how fireflies emit light at different frequencies: once they are brought into contact with other fireflies, their emission of light introduces a coupling that creates large-scale oscillations, as if someone is turning them on and off at the same time, when in fact this is a product of their coupling.

As it happens, a more general form of coupled oscillators, namely non-linear oscillators can also generate turbulence as shown by the Japanese physicist Yoshiki Kuramoto. Building on previous important work starting in the 1940s, Kuramoto was able to show in the 1980s that coupled oscillators can describe turbulence in many different physical systems.

Independently of this research, advances in brain scanning technology in the early 1990s started to generate whole-brain neuroimaging data. In the 2000s we discovered that brain activity can be very accurately described using whole-brain models of coupled oscillators using the anatomical connectivity of the brain. This is much in keeping with Leonardo’s anatomical ambitions, which were ultimately devoted to whole-brain and whole-body functionality in which detailed morphology and local physiology were a means to a more extensive end, namely the characterisation of the human body as a ‘model for the world’.

Merging these streams of research, in 2020 we discovered that not only can these models accurately describe brain dynamics but that this accuracy results from the turbulent fluctuations of brain activity. In other words, the turbulence in the brain is found in the fluctuations of local synchronisation of neural activity rather than the kinetic fluctuations of molecules in fluids. The physical systems are different, but the underlying ordering obeys similar principles, namely that their seemingly chaotic fluctuations are hierarchically ordered, giving rise to turbulence.

Our discovery was based on the key insight from Kuramoto’s independent studies of coupled oscillators for turbulence, where he was able to describe the level of turbulence through a parameter describing the local level of synchronisation. As intuited by Leonardo, turbulence is characterised by the rich variability of different size vortices, which can now be characterised as local synchronised clusters. Kuramoto’s measure captures the vortex space over time, analogous to the rotational vortices found in fluid dynamics, and which Leonardo so carefully characterised in his experiments and synthesised in his drawings.

These discoveries mean that the energy cascade must hold true for brain dynamics too. In fact, the fast, efficient energy transfer in fluids is a key aspect of turbulence. At an abstract level information transfer is analogous to energy transfer, which has been shown by mathematical research that demonstrates close links between the propagation of disturbances and the transmission of information. In the context of brain dynamics, the lens of turbulence provides new tools for analysing and measuring complex phenomena which has hitherto remained hidden. As Leonardo discovered, this lens creates novel perspectives on many things, scientific and artistic.

Following Leonardo, the turbulence framework in modern science is not only a fertile generator for mixing things and discovering order in disorder, but could also be key to understanding the general principles of information processing and transfer in the brain. This is especially true given turbulence’s remarkable success in describing energy transfer in fluids, which has allowed scientists to design better airplanes and optimal methods for mixing chemicals. In fact, it has recently been proposed that the cascade of energy/information into small circuits (forming recoverable whorl patterns) could be a basis of cognition.

As a fertile new framework, the turbulence perspective could thus help solve many important fundamental questions with regards to brain functioning. What are the consequences for information transfer of the underlying turbulent dynamical regime?
in the special architecture offered by the brain? Leonardo was deeply interested in anatomy, conducting many dissections, and injecting a brain with wax to determine the shape of the ventricles. The central ventricle contained both imagination and intellect, working in close conjunction.

The question posed by this new research in living, healthy people suggests that there might be something special about brain architecture that allows for optimal information transfer. For instance, it is clear, perhaps uniquely in nature, that brain architecture contains rare long-range exceptions on top of local coupling following an exponential distance rule. But is this structure and function of these long-range exceptions different across species and could this explain what makes us human? And could this also provide vital clues to how and why brain function breaks down in neuropsychiatric disease? Could this new perspective finally allow for the emergence of personalised medicine?

The eternal search for order in disorder goes on. The artistic vision of Leonardo merged with his scientific insights to capture the vortices of the turbulent fluid dynamics. It would be nice to think, almost half millennium later, that his drawings and search for order in disorder might have inspired later scientists to develop mathematical tools to describe the order in turbulence. Benoît Mandelbrot has actually posited a direct link between Leonardo and Richardson (see video interview https://www.youtube.com/watch?v=ptg7G02c). At least we can be confident that Leonardo would have been excited to see how his seminal ideas are deeply in tune with the modern use of mathematical tools to describe the underlying order of the information cascade in the human brain — and perhaps what ultimately makes us human.

REFERENCES


*Center for Brain and Cognition, Computational Neuroscience Group, Department of Information and Communication Technologies, Universitat Pompeu Fabra, Rec. Bonnac 138, Barcelona, 08018, Spain. 1Institució Catalana de la Recerca i Estudis Avançats (ICREA), Passeig Lluís Companys 23, Barcelona, 08010, Spain. 2Department of Neuropsychology, Max Planck Institute for Human Cognitive and Brain Sciences, 04103 Leipzig, Germany. 3School of Psychological Sciences, Monash University, Melbourne, Clayton VIC 3800, Australia. 4Trinity College, Oxford, University of Oxford, Oxford, UK. 5Centre for Eudaemonic and Human Flourishing, University of Oxford, Oxford, UK. 6Department of Psychiatry, University of Oxford, Oxford, UK. 7Center for Music in the Brain, Department of Clinical Medicine, Aarhus University, Denmark. 8E-mail: morten.kringelbach@psych.ox.ac.uk